Charm quark energy loss in QCD matter

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Abstract. The energy loss of heavy quarks in a quark-gluon plasma of finite size is studied within the light-cone path integral approach. A simple analytical formulation of the radiative energy loss of heavy quarks is derived. This provides a convenient way to quantitatively estimate the quark energy loss. Our results show that if the energy of a heavy quark is much larger than its mass, the radiative energy loss approaches the radiative energy loss of light quarks.

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In high-energy heavy-ion collisions hard scattering of partons occurs in the early stages of the reaction, well before a quark-gluon plasma (QGP) might have been formed, producing fast partons that propagate through the hot and dense medium and lose their energy. Hard hadronic probes have long been thought to detect the formation of a quark-gluon plasma in ultrarelativistic heavy-ion collision. Heavy-quark radiative energy loss is such a probe to study the properties of the quark-gluon plasma. In recent years, the investigation of the parton energy loss in QCD matter has created considerable interest [1–7]. The study of the induced gluon radiation from heavy quarks in QGP matter is of great importance for the understanding of experimental data from high-energy nucleus-nucleus (AA) collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC). Recent measurements [8–13] of high-$p_\perp$ hadron production and its centrality dependence in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV provide the first evidence for medium-induced parton energy loss. The motivation of this paper is to employ the light-cone path integral (LCPI) approach to solve the massive-partons radiative energy loss. The analytical solution is obtained by the way of light-cone path integral. Our analytical result is consistent with the former numerical result —the parton energy loss tends to be smaller for massive quarks than for massless ones.

In 1953 Landau and Pomeranchuk [14] predicted with classical electrodynamics that if the formation length of

the bremsstrahlung becomes comparable to the distance over which the multiple scattering becomes important, the bremsstrahlung will be suppressed. Migdal [15] developed a quantitative theory of this phenomenon. In the current literature, we call the suppression of radiation processes in medium the Landau-Pomeranchuk-Migdal (LPM) effect. First results on the LPM effect in QCD were given by Gyulassy and Wang [16,17]; they first discussed that the parton jet, produced in inelastic collisions, propagating in the QCD matter will lose its energy due to medium-induced gluon radiation (G-W model). They pointed out, comparing with elastic energy loss, that the contribution of inelastic energy loss is more important. The G-W model has been extended by R. Baier, Y. Dokshitzer, A.H. Mueller, S. Peigne and D. Schiff (BDMPS), using equal-time perturbation theory. The calculation of BDMPS about the energy loss of inelastic scattering indicates that the parton radiative energy loss has a square dependence on the path length in the medium [18]. Based on the G-W model, M. Gyulassy, P. Levai and I. Vitev (GLV) developed opacity technology to calculate the jet energy loss [19]. E.K. Wang and X.N. Wang first discussed the detailed balance effect of the jet energy loss in hot QGP medium [3]. They pointed out that the modified energy dependence of the energy loss will affect the suppression shape of moderately high-p_t hadrons due to jet quenching in high-energy heavy-ion collisions. All the above-mentioned approaches are focused on the massless-parton energy loss. Based on the non-Abelian Furry approximation, U.A. Wiedemann obtained a medium-induced gluon distribution [20]. It is a general result and provides a proof of the equivalence of the BDMPS and Zakharov formalisms. They extended

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medium-induced gluon distribution to calculate quenching weights [21] and the massive-quark energy loss [6] and got significative information for jet quenching.

Due to the mass effect, it is hard to solve the problem of massive-parton emission. Up to now, most results of the heavy-quark energy loss have been numerical. The light-cone path integral approach is a simple way of dealing with the bremsstrahlung of photons and gluons and it can give an analytical result. In the path integral formalism, the radiation cross-section is determined by a dipole cross-section which essentially measures the difference between elastic-scattering amplitudes of different projectile Fock state components as a function of impact parameter. Baier, Dokshitzer and Schiff (BDMS) have shown [22] that the evolution of the rescattering amplitude in the BDMS formalism is determined by Zakharov's dipole cross-section [20]. Accordingly, static Debye screened scattering centers are considered and all the scattering centers are supposed to be independent. The probability of gluon emission in the LCPI approach is expressed through the solution of a two-dimensional Schrödinger equation with an imaginary potential. The two-dimensional Hamiltonian reads [23]

$$H = -\frac{1}{2M(x)}\left(\frac{\partial}{\partial \boldsymbol{\rho}}\right)^2 - i\frac{n(z)\sigma_3(\rho,x)}{2}, \qquad (1)$$

where $M(x) = Ex(1-x)$, x is the gluon fractional momentum, $n(z)$ is the number density of the medium at the longitudinal coordinate z and σ_3 is the cross-section of the interaction of a color singlet $q\bar{q}g$ system with a color center.

The contribution of the bremsstrahlung mechanism to the cross-section of gluon production can be written as [7]

$$\frac{\mathrm{d}\sigma_{\mathrm{eff}}^{\mathrm{BH}}(x,z)}{\mathrm{d}x} = \mathrm{Re}\int \mathrm{d}\boldsymbol{\rho}\,\psi^*(\boldsymbol{\rho},x)\sigma_3(\rho,x)\psi(\boldsymbol{\rho},x,z), \quad (2)$$

where $\psi(\boldsymbol{\rho},x)$ is the light-cone wave function for the $q \to qg$ transition in vacuum and $\psi(\boldsymbol{\rho},x,z)$ is the medium-modified light-cone wave function for the $q \to qg$ transition in medium at the longitudinal coordinate z, $\boldsymbol{\rho}$ is the transverse coordinate and x is the Feynman variable of the radiated gluon. The wave functions read [7]

$$\psi(\boldsymbol{\rho},x) = p(x)\left(\frac{\partial}{\partial \rho'_x} - is_g\frac{\partial}{\partial \rho'_y}\right)$$
$$\times \int_0^\infty \mathrm{d}\xi \exp\left(-\frac{i\xi}{L_f}\right)K_0(\boldsymbol{\rho},\xi|\boldsymbol{\rho}',0)|_{\rho'=0}, \quad (3)$$

$$\psi(\boldsymbol{\rho},x,z) = p(x)\left(\frac{\partial}{\partial \rho'_x} - is_g\frac{\partial}{\partial \rho'_y}\right)$$
$$\times \int_0^z \mathrm{d}\xi \exp\left(-\frac{i\xi}{L_f}\right)K_0(\boldsymbol{\rho},z|\boldsymbol{\rho}',z-\xi)|_{\rho'=0}, \quad (4)$$

where $p(x) = i\sqrt{\alpha_s/2x}\,[s_g(2-x) + 2s_q x]/2M(x)$, s_{qg} denotes parton helicities ($s_q = \frac{1}{2}$, $s_g = \pm 1$), K_0 is the Green

function for the two-dimensional Hamiltonian. K_0 can be written as:

$$K_0(\boldsymbol{\rho}_2,z_2|\boldsymbol{\rho}_1,z_1) = \frac{M(x)}{2\pi i(z_2-z_1)}\exp\left[\frac{iM(x)(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1)^2}{2(z_2-z_1)}\right], \qquad (5)$$

where $L_f = 2Ex(1-x)/\epsilon^2$, $\epsilon^2 = m_g^2(1-x) + m_q^2 x^2$, L_f is the gluon formation length. m_q is the quark mass and m_g is the mass of the radiated gluon. The latter plays the role of an infrared cut-off removing contributions of the long-wave gluon excitations which cannot be treated perturbatively. We assume that the heavy quark is produced in the central rapidity region at $\eta = 0$ and the production point is at $z = 0$, propagating through a hard mechanism in a medium of extent L along the z-axis. The induced gluon bremsstrahlung spectrum can be represented as

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \int_0^L \mathrm{d}z\, n(z)\frac{\mathrm{d}\sigma_{\mathrm{eff}}^{\mathrm{BH}}(x,z)}{\mathrm{d}x}, \qquad (6)$$

where $n(z)$ is the number density of the medium.

Using formulas (3)-(5), it is easy to obtain the light-cone wave function after a simple calculation:

$$\psi(\boldsymbol{\rho},x) = \frac{p(x)M^2(x)}{2\pi}(-\rho_x + is_g\rho_y)$$
$$\times \int_0^\infty \frac{\mathrm{d}\xi}{\xi^2}\exp\left(-\frac{i\xi}{L_f}\right)\exp\left[\frac{iM(x)\boldsymbol{\rho}^2}{2\xi}\right], \quad (7)$$

$$\psi(\boldsymbol{\rho},x,z) = \frac{p(x)M^2(x)}{2\pi}(-\rho_x + is_g\rho_y)$$
$$\times \int_0^z \frac{\mathrm{d}\xi}{\xi^2}\exp\left(-\frac{i\xi}{L_f}\right)\exp\left[\frac{iM(x)\boldsymbol{\rho}^2}{2\xi}\right]. \quad (8)$$

Substituting (7) and (8) to (2), the cross-section of gluon production can be represented as

$$\frac{\mathrm{d}\sigma_{\mathrm{eff}}^{\mathrm{BH}}(x,z)}{\mathrm{d}x} = \frac{p^2(x)M^3(x)}{2\pi^2}\mathrm{Re}\int \mathrm{d}\boldsymbol{\rho}\rho\epsilon K_1(\rho\epsilon)i\sigma_3(\rho,x)$$
$$\times \int_0^z \frac{\mathrm{d}\xi}{\xi^2}\exp\left(-\frac{i\xi}{L_f}\right)\exp\left[\frac{iM(x)\boldsymbol{\rho}^2}{2\xi}\right], (9)$$

with $K_1(\rho\epsilon)$ the modified Bessel function of the second kind, $\sigma_3(\rho,x)$ is the three-body cross-section of the imaginary potential [24], $\sigma_3(\rho,x) = C_A/2C_F[\sigma_2((1-x)\rho) + \sigma_2(\rho) - \frac{1}{C_A^2}\sigma_2(x\rho)] = C_3(x)\rho^2$, where $\sigma_2(\rho)$ is the dipole cross-section for scattering of a $q\bar{q}$ pair on a color center, $C_3(x) = C_2(\rho)A(x)$ with $A(x) = [1 + (1-x)^2 - x^2/N_c^2]C_A/2C_F$. In the region $\rho \ll \frac{1}{\mu}$ (here μ is the Debye screening mass), which dominates the spectrum for strong suppression, $C_2(\rho)$ takes the form [7]

$$C_2(\rho) \approx \frac{C_F C_T \alpha_s^2 \pi}{2}\ln(\frac{1}{\rho\mu}), \qquad (10)$$

where C_A, C_F, C_T are the color Casimir operators. From (10), we can see that $C_2(\rho)$ has a slow logarithmic dependence on ρ.

In order to derive a quantitative estimate, we take the charm quark of mass $m_q = 1.5$ GeV. When $\rho\epsilon$ is small, we can expand the Bessel function $K_1(\rho\epsilon)$ and keep the first two terms. After a complex algebra calculation, one can obtain the main contribution of the bremsstrahlung to the cross-section of gluon production:

$$\frac{\mathrm{d}\sigma_{\mathrm{eff}}^{\mathrm{BH}}(x,z)}{\mathrm{d}x} = \frac{\alpha_s^2 C_F C_T A(x) G(x)}{8M(x)}$$
$$\times \left\{ \frac{\pi}{2} L_f \sin \frac{z}{L_f} + L_f(1-c)\left(\cos \frac{z}{L_f} - 1\right) \right.$$
$$\left. + L_f \ln \frac{M(x)}{2\mu^2 z}\left(1 - \cos \frac{z}{L_f}\right) + \frac{z^2}{4L_f}\right\}, \quad (11)$$

where $G(x) = \alpha_s C_F[1-x+x^2/2]/x$ and the Euler constant $c \simeq 0.5772$. The radiative energy loss can be written as

$$\Delta E = \int_{\omega_{\mathrm{cr}}}^{E} \mathrm{d}\omega\omega\frac{\mathrm{d}p}{\mathrm{d}\omega}. \quad (12)$$

Considering light partons, the mass can be neglected, $L_f \to \infty$, $\cos\frac{\xi}{L_f} \approx 1$, $\sin\frac{\xi}{L_f} \approx 0$. From (11) and (12) we obtain the light-quark radiative energy loss ΔE [7]:

$$\Delta E = \frac{C_F \alpha_s}{4} \frac{L^2\mu^2}{\lambda_g} \ln \frac{E}{\omega_{\mathrm{cr}}}, \quad (13)$$
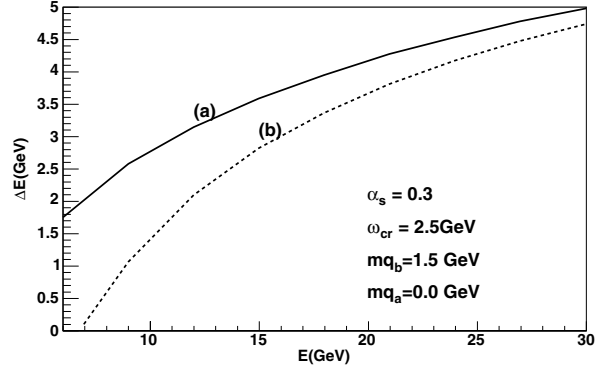
where $\frac{1}{\lambda_g} = \alpha_s^2 \pi C_F C_T A(0)n/2\mu^2$. At $E \to \infty$ the energy loss (13) is equal to GLV's result [19]:

$$\Delta E_{\mathrm{GLV}} = \frac{C_F \alpha_s}{4} \frac{L^2\mu^2}{\lambda_g} \ln \frac{E}{\mu}, \quad (14)$$

where μ is the Debye screening mass and $\omega_{\mathrm{cr}} \sim \max(nC_3L^3/4, L\mu^2/2)$. Formula (14) reflects the logarithm energy dependence of the radiative energy loss.

Now let us discuss the radiative energy loss of a heavy quark with formula (11). Considering the quark mass effect, the gluon formation length L_f is a finite quantity. In formulas (3), (4), (7), (8) and (9), the $\exp(-\frac{i\xi}{L_f})$ is no longer equal to 1 as in the case of a light parton. The mass dependence of the gluon distribution (6) comes from the phase factor $\exp(-\frac{i\xi}{L_f})$, which is analogous to the mass dependence of the gluon distribution which comes from the phase factor $\exp[i\overline{q}(y_l - \overline{y_l})]$, $\overline{q} = x^2 m_q^2/2\omega$ [3]. In the high-energy limit, the formation length, L_f, becomes larger than the quark path length in the QGP, *i.e.* $L_f \gg L$ [16]. Using (6), (11) and (12), we can get

$$\Delta E = \frac{C_F \alpha_s}{4} \frac{L^2\mu^2}{\lambda_g} \left\{ \ln \frac{E}{\omega_{\mathrm{cr}}} \right.$$
$$+ \frac{m_g^2 L}{3\pi\omega_{\mathrm{cr}}}\left(1 - \frac{\omega_{\mathrm{cr}}}{E}\ln\frac{E^2}{2\mu^2 L\omega_{\mathrm{cr}}} + \ln\frac{\omega_{\mathrm{cr}}}{2\mu^2 L}\right)$$
$$\left. + \frac{m_q^2 L}{3\pi E}\left(-\frac{\pi^2}{6} - \frac{\omega_{\mathrm{cr}}}{E}\ln\frac{\omega_{\mathrm{cr}}}{2\mu^2 L} + \ln\frac{E}{2\mu^2 L}\right)\right\}, \quad (15)$$

Fig. 1. The radiative energy loss of light quarks and heavy quarks as a function of their energy in a plasma characterized by $\mu = 0.5$ GeV, $\lambda_g = 1$ fm, $m_g = 0.375$ GeV and $L = 4$ fm. The solid line (a) is for light quarks, which were studied by Zakharov [7]; the dashed line (b) is for heavy quarks studied by the present authors.
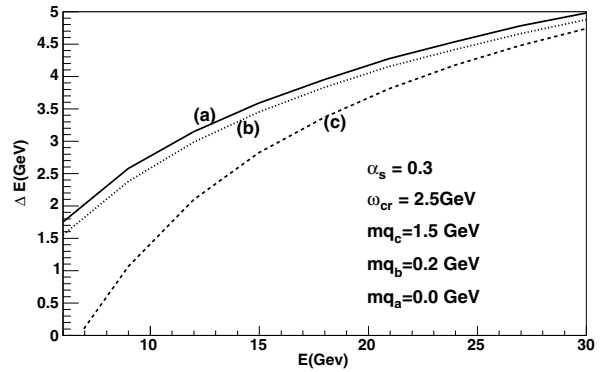
Fig. 2. The radiative energy loss of light quarks and heavy quarks as a function of their energy in a plasma characterized by $\mu = 0.5$ GeV, $\lambda_g = 1.0$ fm, $m_g = 0.375$ GeV and $L = 4$ fm. The solid line (a) is for light quarks, which were studied by Zakharov [7]; the dotted line (b) for a quark of $m_q = 0.2$ GeV; the dashed line (c) is the energy loss for charm quarks via eq. (15). It is shown that ΔE is dependent on the quark mass, the radiative energy loss tends to be smaller for heavy quarks than for light quarks.

where $m_g = 0.375$ GeV is the mass of the related gluon. From (15) we can see that the first term is the radiative energy loss of a light quark. Considering the quark mass effect, the latter terms are the modifications of the light quark.

For the sake of seeing the suppression of the heavy-quark energy loss more clearly, we use fig. 1 and fig. 2 to show the radiative energy loss of light quarks and heavy quarks in a QGP and the dependence on the mass. Figure 1 shows the radiative energy loss of light quarks and heavy quarks. In fig. 1 we can see that the difference between light quarks and heavy quarks becomes smaller with increasing energy of the quark. It is obvious that when the quark energy is much larger than its mass, the

heavy-quark energy loss is approaching the one of light quarks. At high energies, our results are consistent with Gyulassy's calculations [25]. To illustrate the quark mass dependence of ΔE, we use fig. 2 to compare the results for a heavy quark (charm quark $m_q = 1.5$ GeV) to light quarks with mass $m_q = 0$ and $m_q = 0.2$ GeV. With the above values of m_q, one can see the extent of the ΔE dependence on the quark mass. Both fig. 1 and fig. 2 show that the formulas (13), (14) and (15) are applicable in the high-energy limit. At RHIC ($\sqrt{s_{NN}} = 200$ GeV) and LHC ($\sqrt{s_{NN}} = 5.5$ TeV) energies, all the above formulas are operable.

In this work, we assume static Debye screened scattering centers and that all the centers are independent. The energy of the parton is supposed to be high enough so that the condition $L_f \gg L$ is fulfilled. We use the light-cone path integral method to deal with the gluon emission and thus obtain a simple analytical formula for the heavy-quark radiative energy loss. Equation (15) can be used to estimate the measurable yield of hadrons containing heavy quarks. To our knowledge, up to now most of the calculations of the heavy-quark radiative energy loss have been carried out by numerical simulations. In the high-energy limit our results are consistent with Gyulassy's numerical calculations [25]. But one should keep in mind that when the quark mass becomes larger, $\rho\epsilon$ is no longer a small quantity and the Bessel function $K_1(\rho\epsilon)$ cannot be expanded as above. In a following work, we will try to extend our result to accommodate all quark masses.

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